EC 3210 Solutions

Assignment 5

4.1. Given a coefficient B_{ij} of 10^{19} m^3·W^{-1}·s^{-3} for a 650 nm transition in a material with $n = 1$,

a. Calculate A$_{ji}$ and the spontaneous lifetime of the transition.

b. Calculate the ratio of A$_{ij}$ to the stimulated transition rate if the irradiance is 10 mW/mm^2 concentrated at the transition wavelength and $g(\nu_0) = 10^{-10}$.

a. The desired quantities are

$$A_{ij} = \frac{8\pi n^3 h}{\lambda^3} B_{ij} = \left(\frac{8\pi (1)^3 6.63 \times 10^{-34}}{(650 \times 10^{-9})^3} \right) (10^{19}) = 6.06 \times 10^5 \text{ s}^{-1} \tag{1}$$

and

$$\tau_s = \frac{1}{A_{ij}} = \frac{1}{6.06 \times 10^5} = 1.649 \times 10^{-6} \text{ s}\,. \tag{2}$$

b. We note that $I = 10 \times 10^{-3}$ W·mm^{-2} is equal to 10×10^3 W·m^{-2}, and, so,

$$\begin{aligned} \frac{A_{ij}}{B_{ij}\rho_\nu} &= \frac{A_{ij}}{B_{ij}\left(\dfrac{nI\delta(\nu - \nu_0)}{c} \right) g(\nu)} = \frac{A_{ij}c}{B_{ij} n I(\nu_0) g(\nu_0)} \\ &= \frac{(6.06 \times 10^5)(3.0 \times 10^8)}{(1 \times 10^{19})(1)(1 \times 10^4)(1 \times 10^{-10})} = 18.17\,. \end{aligned} \tag{3}$$

4.2 The loss coefficient of a material is 1% per mm.

a. Calculate the fraction of the power transmitted through a 5 cm thickness of the material.

b. Find the value of the loss coefficient α.

a. We find

$$\alpha = \frac{0.01}{0.001} = 10 \text{ m}^{-1} \tag{4}$$

$$\frac{I_{\text{out}}}{I_{\text{in}}} = e^{-\alpha D} = e^{-(10)(5 \times 10^{-2})} = 0.607 = 60.7\%\,. \tag{5}$$

b. We have already found this value. From above, $\alpha = 10$ m^{-1}.

4.3 A material exhibits a gain of 1% per meter when pumped at a certain power level.

a. Calculate the gain coefficient β.

b. Find the length of material required to double the power in a wave.

a. We find that, for 1 meter,

$$\beta \approx \frac{\alpha L}{L} = \frac{(0.01)(1)}{1} = 0.01 \text{ m}^{-1}\,. \tag{6}$$

b. To double the power requires

$$G = e^{+\beta z} = 2\,, \tag{7a}$$

so

$$\ln 2 = \beta z \tag{7b}$$

and

$$z = \frac{\ln 2}{\beta} = \frac{\ln 2}{0.01} = 69.3\ \mathrm{m}\,. \tag{7c}$$

4.4 A ruby laser operates at 694 nm from an upper level to the ground state.

a. Calculate the temperature that is required to place 50% of the total number of atoms in the upper state (leaving 50% in the ground state).

b. Calculate the temperature that is required to place 10% of the total number of atoms in the upper state (leaving 90% in the ground state).

c. Calculate the fraction of the total population that are in the upper state when T is 300K.

See Fig. 1 for the population levels.

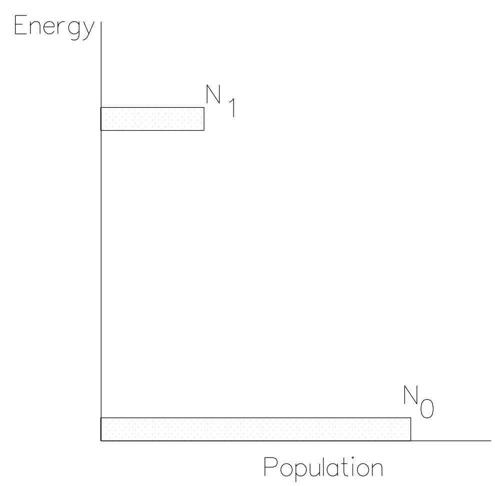

Figure 1: Energy levels and populations for Problem 4.4

a. The temperature required to place 50% of the total atoms in the upper state (leaving 50% in the lower state) is found from

$$\frac{N_1}{N_0} = \exp\left(-\frac{\Delta E}{kT}\right) = 1\,, \tag{8a}$$

so

$$-\frac{\Delta E}{kT} = 0 \tag{8b}$$

and

$$T \to \infty\,. \tag{8c}$$

It would take an infinite temperature to get 50% of the atoms into the upper level *if the material were in thermal equilibrium.*

b. To place 10% in the upper level would require

$$\frac{N_1}{N_0} = \exp\left(-\frac{\Delta E}{kT}\right) = \frac{0.1}{0.9}\,, \tag{9a}$$

so

$$-\frac{\Delta E}{kT} = \ln\left(\frac{1}{9}\right) = -2.20 \tag{9b}$$

and

$$T = -\frac{\Delta E}{k(-2.20)} = \frac{hc}{\lambda k(2.20)} = 9.42 \times 10^3 \text{ K}\,. \tag{9c}$$

c. The fraction of the *total* population that is in the upper stage when $T = 300$ K is

$$\frac{N_1}{N_{\text{total}}} = \frac{N_1}{N_0 + N_1} = \frac{1}{1 + \dfrac{N_0}{N_1}}\,, \tag{10a}$$

where

$$\begin{aligned} \frac{N_0}{N_1} &= \frac{1}{\dfrac{N_1}{N_0}} = \exp\left(\frac{\Delta E}{kT}\right) = \exp\left(\frac{hc}{\lambda kT}\right) \qquad (10b) \\ &= \exp\left(\frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{(694 \times 10^{-9})(1.38 \times 10^{-23})(300)}\right) = 1.161 \times 10^{30} \end{aligned}$$

and, so,

$$\frac{N_1}{N_{\text{total}}} = \frac{1}{1 + \frac{N_0}{N_1}} = \frac{1}{1 + 1.161 \times 10^{30}} = 8.61 \times 10^{-31}\,. \tag{10c}$$

Hence, we find that the upper level is *really* empty at thermal equilibrium at room temperature.

4.5 Calculate the population inversion required per unit volume to give a small–signal gain coefficient of 0.5 m^{-1} in a CO_2 laser. Assume that A_{ij} is 200 s^{-1}, $g(\nu_0) \approx 1/\Delta\nu = 1 \times 10^{-9}$ s, and $n = 1$.

First we note that

$$\beta = \frac{(N_j - N_i)B_{ij} nh\nu_0 g(\nu_o)}{c\text{Vol}}\,, \tag{11}$$

so

$$\frac{N_j - N_i}{\text{Vol}} = \frac{\lambda\beta}{B_{ij} n h g(\nu_0)} \, . \tag{12}$$

We also note that

$$B_{ij} = \frac{\lambda^3 A_{ij}}{8\pi n^3 h} = \frac{(10.6 \times 10^{-3}(200)}{8\pi(1)^3(6.63 \times 10^{-34})} = 1.43 \times 10^{19} \tag{13}$$

and, so,

$$\begin{aligned} \frac{N_j - N_i}{\text{Vol}} &= \frac{(10.6 \times 10^{-6})(0.5)}{(1.43 \times 10^{19})(1)(6.64 \times 10^{-34})(1 \times 10^{-9})} \\ &= 5.59 \times 10^{17} \text{ molecules} \cdot \text{m}^{-3} \, . \end{aligned} \tag{14}$$